B.B.A 1st year 11nd SEM

Business Statistics

unit - III

Index Numbers

[Continuation]

1. Weighted average of price Relative using Arithmetic mean:

$$P_{01}(A.M) = \frac{\Sigma WP}{\Sigma W} \quad ; \text{ where}$$

$$P = \frac{P_1}{P_0} \times 100 = \text{Price Relative}$$

and W = weights.

NOTE :- If weights are not given then take the base year Quantities (q_0) as weights.

2. weighted average of price Relative using Geometric mean:

$$P_{01}(G.M) = \text{Antilog}\left[\frac{\Sigma W \log P}{\Sigma W}\right] \text{ where,}$$

$$P = \frac{\text{current year price}}{\text{Base year price}} \times 100 = \frac{P_1}{P_0} \times 100$$

and W = weights.

Pro

Problem : An enquiry into the budgets of middle2 class families in a city gave the following information :

Expenses on :	Food	Rent	clothing	Fuel	others
	30%	15%	20%	10%	25%
Price (in Rs.) in 2002 :	100	20	70	20	40
Price (in Rs.) in 2005 :	90	20	60	15	55

~~solution~~ :- Compute the price Index number using

(i) weighted A.M of price relatives and

(ii) weighted G.M of price relatives.

Solution :- Computation of Price Index using A.M & G.M

Items	weights (w)	Prices		Price Relative $P = \frac{P_1}{P_0} \times 100$	wp
		In 2002 (P_0)	In 2005 (P_1)		
Food	30	100	90	$\frac{90}{100} \times 100 = 90$	$30 \times 90 = 2,700$
Rent	15	20	20	$\frac{20}{20} \times 100 = 100$	$15 \times 100 = 1500$
clothing	20	70	60	$\frac{60}{70} \times 100 = 85.7$	1,714
fuel	10	20	15	$\frac{15}{20} \times 100 = 75$	750
others	25	40	55	$\frac{55}{40} \times 100 = 137.5$	3,437.5
$\Sigma w = 100$					$\Sigma wp = 10101.5$

log P	w log P
log 90 = 1.9542	30 × 1.9542 = 58.6260
log 100 = 2	15 × 2 = 30
log 85.7 = 1.9330	20 × 1.9330 = 38.6600
log 75 = 1.8751	10 × 1.8751 = 18.7510
log 137.5 = 2.1383	25 × 2.1383 = 53.4570
	$\Sigma w \log P = 199.4940$

(i) Index Number based on weighted Arithmetic mean of Price Relative is given by

$$P_{01}(A.M) = \frac{\Sigma w P}{\Sigma w} = \frac{10101.5}{100} = 101.015$$

$$\therefore P_{01}(A.M) = 101.015$$

(ii) Index Number based on weighted Geometric mean of Price Relative is given by

$$P_{01}(G.M) = \text{Antilog}\left[\frac{\Sigma w \log P}{\Sigma w}\right]$$

$$= \text{Antilog}\left[\frac{199.494}{100}\right]$$

$$= \text{Antilog}(98.83)$$

$$P_{01}(G.M) = 98.83$$

Tests of Consistency of Index Numbers.

1. Time Reversal Test :- The time reversal test, proposed by prof. Irving Fisher. The formula for calculating an index Number should be such that it gives the same ratio between one point of comparision and the other, no matter which of the two is taken as the base or putting it another way, the index number reckoned forward should be reciprocal of the one reckoned backward.

$$\text{i.e., } P_{01} \times P_{10} = 1$$

where,

P_{01} is the price index for the current year '1' with respect to the base year '0', (forward).

P_{10} is the price index for the current year '0' withrespect to the base year '1' (Backward) and.

2. Factor Reversal Test :- The factor Reversal test was proposed by prof. Irving Fisher.

If the price and quantity indices are obtained for the same data, same base and current periods and using the same formula then their product should give the true value ratio since the

multiplied by quantity gives total value. we should have (without factor 100)

$$P_{01} \times Q_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0} = V_{01} = \text{Value}$$

Price × Quantity = Total value.

NOTE :- Fisher's index satisfies both Time Reversal test and Factor Reversal test.

Problem :- For the following data prove that the Fisher's ideal index satisfies both the Time Reversal Test and the Factor Reversal Test. and calculate it value.

Commodity	Base year		Current year	
	Price	Quantity	Price	Quantity
A	6	50	10	56
B	2	100	2	120
C	4	60	6	60
D	10	30	12	24

Solution :- Computation of Fisher's Index

Commodity	P_0	q_0	P_1	q_1	P_0q_0	P_0q_1	P_1q_0	P_1q_1
A	6	50	10	56	300	336	500	560
B	2	100	2	120	200	240	200	240
C	4	60	6	60	240	240	360	360
D	10	30	12	24	300	240	360	288
					$\Sigma P_0q_0 =$ 1040	$\Sigma P_0q_1 =$ 1056	ΣP_1q_0 1420	ΣP_1q_1 1448

Fisher's Price Index number is given by

$$P_{01}^{F} = \sqrt{P_{01}^{L} \times P_{01}^{Pa}}$$ where,

$$P_{01}^{L} = \frac{\Sigma P_1q_0}{\Sigma P_0q_0} \times 100 \qquad P_{01}^{Pa} = \frac{\Sigma P_1q_1}{\Sigma P_0q_1} \times 100$$

$$= \frac{1420}{1040} \times 100 \qquad = \frac{1448}{1056} \times 100$$

$$P_{01}^{L} = 1.3654 \times 100 \qquad P_{01}^{Pa} = 1.37121 \times 100$$

$$P_{01}^{L} = 136.54 \qquad P_{01}^{Pa} = 137.12$$

$$\therefore P_{01}^{F} = \sqrt{136.54 \times 137.12}$$

$$P_{01}^{F} = \sqrt{18.728 \cdot 3648}$$

$$\therefore P_{01}^{F} = 136.8297$$

Time Reversal Test :-

$$P_{01}^{F} \times P_{10}^{F} = 1 \quad \text{(without Factor 100)}$$

$$P_{01}^{F} \times P_{10}^{F} = \sqrt{P_{01}^{L} \times P_{10}^{Pa}} \times \sqrt{P_{10}^{L} \times P_{10}^{Pa}}$$

$$= \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times \frac{\Sigma P_0 q_1}{\Sigma P_1 q_1} \times \frac{\Sigma P_0 q_0}{\Sigma P_1 q_0}}$$

$$= \sqrt{\frac{1420}{1040} \times \frac{1448}{1056} \times \frac{1056}{1448} \times \frac{1040}{1420}}$$

$$= \sqrt{1}$$

$$P_{01}^{F} \times P_{10}^{F} = 1$$

Hence, Fisher's Index satisfies Time Reversal Test.

Factor Reversal Test :-

$$P_{01}^{F} \times Q_{01}^{F} = V_{01}$$

Price x Quantity = Value.

$$P_{01}^{F} \times Q_{01}^{F} = \sqrt{P_{01}^{L} \times P_{01}^{Pa}} \times \sqrt{Q_{01}^{L} \times Q_{01}^{Pa}}$$

$$= \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times \frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}}$$

$$= \sqrt{\frac{1420}{1040} \times \frac{1448}{1056} \times \frac{1056}{1040} \times \frac{1448}{1420}}$$

$$= \sqrt{\left(\frac{1448}{1040}\right)^2}$$

$$P_{01}^{F} \times Q_{01}^{F} = \frac{1448}{1040} = 1.3923 \quad \text{---} \quad (1)$$

Value Index :- (without Factor 100)

$$V_{01} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0} = \frac{1448}{1040} = 1.3923 \quad (2)$$

From equations (1) and (2) we write

$$P_{01}^{F} \times Q_{01}^{F} = V_{01}$$

Hence, ~~Factor~~ Fisher's index satisfies Factor Reversal Test.

Unweighted Index Number :-

Unweighted Index Number can be divided into two types.

① Simple Aggregate method.

② Simple Averages of price Relatives.

1. Simple Aggregate method :- This is the simplest of all the methods of constructing Index numbers and consists in expressing aggregate of prices in the current year as a percentage of the aggregate of prices in the base year. Thus the price index for the current year with respect to the base year is given by

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100.$$

2. Simple Average of price Relative method :-

In this method, the price of each item in the current year is expressed as a percentage of its price in the base year. The figure obtained is called price Relative. For calculation of price relative we use either Arithmetic mean or Geometric mean it is given by

$$P_{01}(A.M) = \frac{\Sigma \frac{P_1}{P_0} \times 100}{n} = \frac{\Sigma P}{n}$$

where n = number of items.

Problem : From the following data construct an index number for 1994 by taking 1993 as base by using

① Simple Aggregate method

② Simple Average of Price Relative method.

Solution: Computation of ~~Index~~ index number

Commodities	Price in 1993	Price in 1994
A	40	60
B	60	90
C	85	125
D	25	35
E	30	40

Solution : Computation of an Index Number

Commodities	Price in 1993 (P_0)	Price in 1994 (P_1)	Price Relative $P = \frac{P_1}{P_0} \times 100$
A	40	60	$\frac{60}{40} \times 100 = 150$
B	60	90	$\frac{90}{60} \times 100 = 150$
C	85	125	$\frac{125}{85} \times 100 = 147.0588$
D	25	35	$\frac{35}{25} \times 100 = 140$
E	30	40	$\frac{40}{30} \times 100 = 133.333$
	$\Sigma P_0 = 240$	$\Sigma P_1 = 350$	$\Sigma P = 720.3921$

① Simple Aggregate method :

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100 = \frac{350}{240} \times 100 = 145.8333$$

$$\therefore P_{01} = 145.8333$$

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② Simple Average of price Relative method.

$$P_{01}(A.M) = \frac{\Sigma P}{n} = \frac{720.3921}{5} = 144.0784$$

$$P_{01}(A.M) = 144.0784$$

IV

Fixed Based Index Numbers :- Index Numbers constructed with some particular fixed reference period (i.e., base period) is known as fixed Based Index Numbers.

Chain Based Index Numbers :- The series of index numbers computing for each period with preceding period as base period are called Chain Based Index Numbers.

The steps in the construction of the chain base index numbers may be summarised as follows.

→ Calculate Link Relative (L.R) for the given data.

i.e., L.R of current year $= \frac{\text{current year price}}{\text{previous year price}} \times 100$.

→ Now the chain based index numbers are obtained by the following formula.

C.B.I for any year $= \frac{\text{current year L.R} \times \text{Preceding year C.B.I}}{100}$

NOTE : C.B.I numbers for ~~base~~ first period is ~~always~~ 100. Same as the Index Numbers for the first period.

Conversion of Chain Based Index numbers into Fixed Based Index Numbers:-

Current year F.B.I $= \frac{\text{current year C.B.I} \times \text{Previous year F.B.I}}{100}$

NOTE : F.B.I for the first period being same as the C.B.I for the first period.

Problem:- From the chain base index numbers given below find fixed base index numbers.

year	: 1995	1996	1997	1998	1999
Chain base index:	80	110	120	90	140

Solution:- we know that

$$\text{current year F·B·I} = \frac{\text{current year C·B·I} \times \text{previous year F·B·I}}{100}$$

Conversion of C·B·I numbers to F·B·I numbers

Year	chain Base Index Number	Fixed Base Index Number
1995	80	80
1996	110	$\frac{80 \times 110}{100} = 88$
1997	120	$\frac{88 \times 120}{100} = 105.60$
1998	90	$\frac{105.6 \times 90}{100} = 95.04$
1999	140	$\frac{95.04 \times 140}{100} = 133.06$

Problem: Calculate the chain Based index Number for the following data.

Year : 1990	1991	1992	1993	1994	1995
Index : 376	392	408	380	392	400

Solution:-

Calculation of C.B.I

Year	Index	Link Relatives	Chain Based Index
1990	376	----	376
1991	392	$\frac{392}{376} \times 100 = 104.26$	$\frac{376 \times 104.26}{100} = 392$
1992	408	$\frac{408}{392} \times 100 = 104.08$	$\frac{392 \times 104.08}{100} = 408$
1993	380	$\frac{380}{408} \times 100 = 93.14$	$\frac{408 \times 93.14}{100} = 380$
1994	392	$\frac{392}{380} \times 100 = 103.16$	$\frac{380 \times 103.16}{100} = 380$
1995	400	$\frac{400}{392} \times 100 = 102.04$	$\frac{392 \times 102.04}{100} = 400$

Base shifting :- Base shifting means the changing of the given base year of a series of index numbers and recasting them into a new series based on some recent new base year.

Shifting of base period is needed in the following situations.

1. when the base year is too old or too distant from the current period to make meaningful and valid comparisons.
2. If we want to compare series of index numbers with different base periods, to make

quick and valid comparisons both the series must be expressed with a common base year.

The index number of the new base year as 100 and then expressing the given series of index numbers as a percentage of the index number of the time period selected as the new base year.

Thus the series of index numbers with new base year is calculated by using the following formula.

$$\text{Recast Index number of any year} = \frac{\text{Old Index numbers of the year}}{\text{Index numbers of new base year}} \times 100$$

$$= \frac{100}{\text{Index NO. of new base year}} \times \text{Old Index NO. of the year.}$$

Problem: Reconstruct the following indices using 2000 as the base.

Year :	1996	1997	1998	1999	2000	2001	2002
Index NO's :	110	130	150	175	180	200	220

Solution:- Index Numbers (Base year 2000 = 100) is

Year	Index Number	Index Number (Base 2000 = 100)
1996	110	$\frac{100}{180} \times 110 = 61.11$
1997	130	$\frac{100}{180} \times 130 = 72.22$
1998	150	$\frac{100}{180} \times 150 = 83.33$
1999	175	$\frac{100}{180} \times 175 = 97.22$
2000	180	$\frac{100}{180}$
2001	200	$\frac{100}{180} \times 200 = 111.11$
2002	220	$\frac{100}{980} \times 220 = 122.22$

VI

Cost of Living Index Numbers :-

The cost of living Index numbers are constructed by the following methods.

① Aggregate Expenditure method or weighted Aggregate method: In this method the quantities consumed in the base year are considered as weights. Thus in the usual notations

$$\text{Cost of Living Index} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$$

$$= \frac{\text{Total Expenditure in current year}}{\text{Total Expenditure in Base year}} \times 100$$

② Family Budget method or method of weighted Relatives : In this method the cost of living Index is obtained on taking the weighted average of price - relatives, the weights being the values of the quantities consumed in the base year.

Thus if we write,

$$\text{Cost of Living Index} = \frac{\Sigma PW}{\Sigma W} \quad \text{or} \quad \frac{\Sigma IW}{\Sigma W}$$

where; P or I = Price Relative $= \frac{P_1}{P_0} \times 100$

and $W = P_0 q_0$ = Total value in the base year.

Problem : An enquiry into the budgets of the middle class families in a certain city in india gave the following information:

Expenses on	Food 35%	Fuel 10%	Clothing 20%	Rent 15%	Others 20%
Price in 1995 (Rs.)	150	25	75	30	40
price in 1996 (Rs.)	145	23	65	30	45

What is the cost of living Index of 1996 as compared with that of 1995

Solution :-

Group	weights (w)	price in 1995 P_0	price in 1996 P_1	Price Relative P	PW
Food	35	150	145	$\frac{145}{150} \times 100 = 96.6667$	3383.334
Fuel	10	~~145~~ 25	23	$\frac{23}{25} \times 100 = 92$	920
Clothing	20	75	65	$\frac{65}{75} \times 100 = 86.6667$	1733.334
Rent	15	30	30	$\frac{30}{30} \times 100 = 100$	1500
Others	20	40	45	$\frac{45}{40} \times 100 = 112.5$	2250
	$\Sigma w = 100$				9786.6685

$$\text{Cost of Living Index} = \frac{\Sigma PW}{\Sigma w} = \frac{9786.6685}{100}$$

(family Budget method)

Cost of Living Index = 97.8667

Problem : construct the cost of living index number from the table given below.

Group	Index for 1998	weight
Food	550	46
clothing	215	10
Fuel and Lighting	220	7
House Rent	150	12
Others	275	25

Computation of cost of Living Index number.

Group	Index (I)	weight (w)	WI
Food	550	46	25300
Clothing	215	10	2150
Fuel & Lighting	220	07	1540
House Rent	150	12	1800
Others	275	25	6875
		$\Sigma w = 100$	$\Sigma WI = 37665$

Cost of Living index number for 1998 (Family Budget method) $= \frac{\Sigma WI}{\Sigma w} = \frac{37665}{100} = 376.65$

$\therefore$ Cost of Living Index number for 1998 $= 376.65$

Problem: Compute Cost of Living Index number by Aggregate expenditure method.

Commodity	Quantity consumed in Base year	price in Base year	price in current year
Rice	50	15	30
wheat	4	10	18
pulses	3	30	85
sugar	5	16	28
clothing	10	15	27

Solution:

Computation of Cost of Living Index by Aggregat Expenditure method. 20

Commodity	q_0	P_0	P_1	P_0q_0	P_1q_0
Rice	50	15	30	750	1500
wheat	4	10	18	40	72
Pulses	3	30	85	90	255
Sugar	5	16	28	80	140
Clothing	10	15	27	150	270
				$\Sigma P_0q_0 = 1110$	$\Sigma P_1q_0 = 2237$

$$\text{Cost of Living Index} = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$$

$$= \frac{2237}{1110} \times 100$$

$$\text{Cost of Living Index} = 201.5315$$